Student Workbook

Binomial Theorem

Name: ________________________________ Date: _________________ Class: _________

ESSENTIAL QUESTION

How can Pascal's Triangle and the Binomial Theorem help us efficiently expand powers of binomials?

Lesson Overview

The Binomial Theorem gives a formula for expanding any power of a binomial $(a + b)^n$ without multiplying it out step by step. The coefficients in each expansion come directly from Pascal's Triangle or from binomial coefficients $C(n, k) = n! / [k!(n - k)!]$.

$$(a+b)^n = \Sigma(k=0 \text{ to } n)\ C(n,k)\cdot a^{n-k}\cdot b^k$$
$$\text{Term } (k+1) = C(n,k)\cdot a^{n-k}\cdot b^k$$

Binomial	A polynomial with exactly two terms, e.g. $(a + b)$.
Binomial Coefficient C(n,k)	The number of ways to choose k items from n: $C(n,k) = n!/[k!(n-k)!]$.
Binomial Theorem	$(a+b)^n = \Sigma(k=0 \text{ to } n)\ C(n,k)\cdot a^{n-k}\cdot b^k$ — gives every term of the expansion.
General Term	The (k+1)th term of $(a+b)^n$ is $C(n,k)\cdot a^{n-k}\cdot b^k$.

Worked Examples

EXAMPLE 1

Expand $(x + y)^4$ using the Binomial Theorem.

- Identify: $a = x$, $b = y$, $n = 4$. Write the sum: $\Sigma(k=0 \text{ to } 4)\ C(4,k)\cdot x^{4-k}\cdot y^k$.
- $k=0$: $C(4,0)\cdot x^4\cdot y^0 = x^4$; $k=1$: $C(4,1)\cdot x^3\cdot y = 4x^3y$; $k=2$: $C(4,2)\cdot x^2\cdot y^2 = 6x^2y^2$
- $k=3$: $C(4,3)\cdot x\cdot y^3 = 4xy^3$; $k=4$: $C(4,4)\cdot y^4 = y^4$

Answer: $(x + y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

EXAMPLE 2

Expand $(2x - 3)^3$.

- Rewrite as $(2x + (-3))^3$. So $a = 2x$, $b = -3$, $n = 3$.
- $k=0$: $(2x)^3 = 8x^3$; $k=1$: $C(3,1)(2x)^2(-3) = 3\cdot 4x^2\cdot(-3) = -36x^2$
- $k=2$: $C(3,2)(2x)(-3)^2 = 3\cdot 2x\cdot 9 = 54x$; $k=3$: $(-3)^3 = -27$

Answer: $(2x - 3)^3 = 8x^3 - 36x^2 + 54x - 27$

EXAMPLE 3

Find the 4th term of $(x + 2)^6$.

- The (k+1)th term formula: $C(n,k)\cdot a^{n-k}\cdot b^k$. 4th term → k+1=4 → k=3.
- C(6,3) = 6!/(3!·3!) = 720/(6·6) = 20
- Term = $C(6,3)\cdot x^{6-3}\cdot 2^3 = 20\cdot x^3\cdot 8$

Answer: The 4th term of $(x + 2)^6$ is $160x^3$.

EXAMPLE 4

Find the coefficient of x^3y^2 in $(x + y)^5$.

- $(x+y)^5 = \Sigma(k=0 \text{ to } 5)\ C(5,k)\cdot x^{5-k}\cdot y^k$. We need x^3y^2, so 5−k=3 → k=2.
- Coefficient = C(5,2) = 5!/(2!·3!)

Answer: The coefficient of x^3y^2 in $(x + y)^5$ is 10.

EXAMPLE 5

Use Pascal's Triangle to expand $(a + b)^5$.

- Row 5 of Pascal's Triangle: 1, 5, 10, 10, 5, 1 — the coefficients for k=0..5.
- k=0: a^5; k=1: $5a^4b$; k=2: $10a^3b^2$; k=3: $10a^2b^3$; k=4: $5ab^4$; k=5: b^5

Answer: $(a + b)^5 = a^5 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5ab^4 + b^5$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

1 Expand $(x + 1)^4$ using the Binomial Theorem.

Hint: Use a = x, b = 1, n = 4. Row 4 of Pascal's Triangle is 1, 4, 6, 4, 1.

2 Expand $(x - 2)^3$.

Hint: Rewrite as $(x + (-2))^3$. Apply the Binomial Theorem with a = x, b = −2, n = 3. Watch the signs!

3 Find the 3rd term of $(x + 3)^5$.

Hint: 3rd term → k = 2. Use $C(5,2)\cdot x^{5-2}\cdot 3^2$.

4 Find the coefficient of x^2y^3 in $(x + y)^5$.

Hint: Set 5 − k = 2, so k = 3. The coefficient is C(5, 3).

5 Write row 6 of Pascal's Triangle.

Hint: Start with 1. Each entry is the sum of the two entries directly above it in row 5: 1, 5, 10, 10, 5, 1.

Key Vocabulary

Binomial	A polynomial with exactly two terms, e.g. (a + b).
Pascal's Triangle	A triangular array where each entry is the sum of the two entries above it; rows give binomial coefficients.
Binomial Coefficient C(n,k)	The number of ways to choose k items from n: C(n,k) = n!/[k!(n−k)!].
Binomial Theorem	$(a+b)^n = \Sigma(k=0 \text{ to } n)\ C(n,k)\cdot a^{n-k}\cdot b^k$ — gives every term of the expansion.
General Term	The (k+1)th term of $(a+b)^n$ is $C(n,k)\cdot a^{n-k}\cdot b^k$.

Quick Check Quiz

Circle the letter of the correct answer.

1 **Which row of Pascal's Triangle gives the coefficients for $(a + b)^4$?** Multiple Choice

A Row 3: 1, 3, 3, 1
B Row 4: 1, 4, 6, 4, 1
C Row 5: 1, 5, 10, 10, 5, 1
D Row 2: 1, 2, 1

Common Mistakes to Avoid

✗ Writing $(a + b)^3 = a^3 + b^3$ (forgetting middle terms).

✓ $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$ — always use the Binomial Theorem or Pascal's Triangle.

✗ Forgetting to raise b to the correct power when b is negative, e.g. $(-3)^2 = -9$.

✓ $(-3)^2 = +9$. Negative numbers raised to even powers are positive.

✗ Using k instead of k+1 when identifying the term number (off-by-one error).

✓ The (k+1)th term corresponds to index k. For the 4th term, k = 3.

✗ Computing C(n,k) as n!/k! instead of n!/[k!(n−k)!].

✓ Always divide by both k! and (n−k)! in the binomial coefficient formula.

Math Tips

★ Pascal's Triangle is fastest for small n ($n \le 6$); use the formula C(n,k) for larger n.

★ The exponents of a decrease from n to 0 while the exponents of b increase from 0 to n — they always sum to n.

★ The expansion of $(a+b)^n$ has exactly n+1 terms.

★ The coefficients are symmetric: C(n,k) = C(n, n−k).

★ To find a specific term quickly, use the general term formula instead of expanding fully.